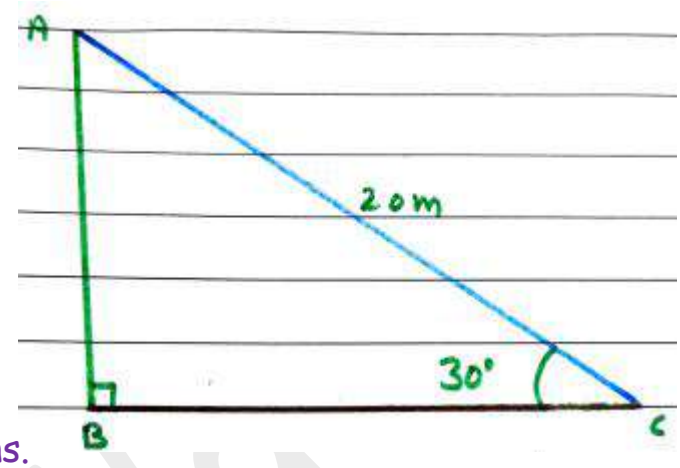
Exercise 9.1

1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.

Sol. Let AB is the length of rope and A is the observation point.



In $\perp \Delta$ ABC

$\frac{BC}{AC} = \sin\theta$

$\Rightarrow BC = AC \times \sin 30°$

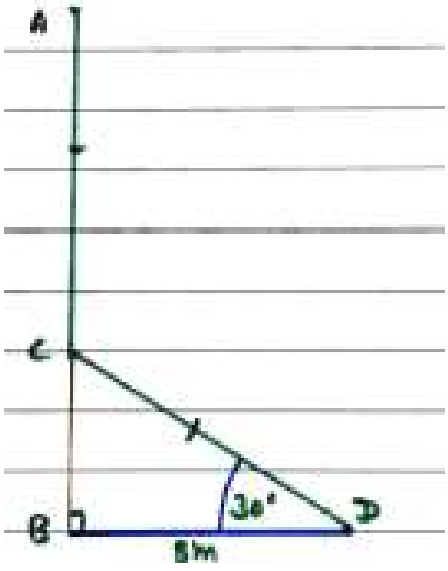
$\Rightarrow BC = 20 \times \frac{1}{2}$

$\Rightarrow BC = 10$

$\therefore$ The height of the pole is 10 m Ans.

2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Sol. Let AB is the height of the tree and C is the broken point.



In $\perp \Delta$ BCD, $\frac{BC}{BD} = \tan\theta$

$\Rightarrow BC = BD \times \tan 30°$

$\Rightarrow BC = 8 \times \frac{1}{\sqrt{3}}$

$\Rightarrow BC = 8 \times \frac{\sqrt{3}}{3}$

$\Rightarrow BC = \frac{8\sqrt{3}}{3}$

And $\frac{BD}{CD} = \cos\theta$

$\Rightarrow \frac{8}{CD} = \cos 30°$

$\Rightarrow \frac{8}{CD} = \frac{\sqrt{3}}{2}$

$\Rightarrow 16 = CD \times \sqrt{3}$

$\Rightarrow CD = \frac{16}{\sqrt{3}}$

$\Rightarrow CD = \frac{16}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow CD = \frac{16\sqrt{3}}{3}$

Height of tree = AC + BC

= CD + BC [$\because$ AC = CD]

$= \frac{16\sqrt{3}}{3} + \frac{8\sqrt{3}}{3}$

$= \frac{24\sqrt{3}}{3}$

$= 8\sqrt{3}$ m

$\therefore$ The height of the tree is $8\sqrt{3}$ m. Ans.

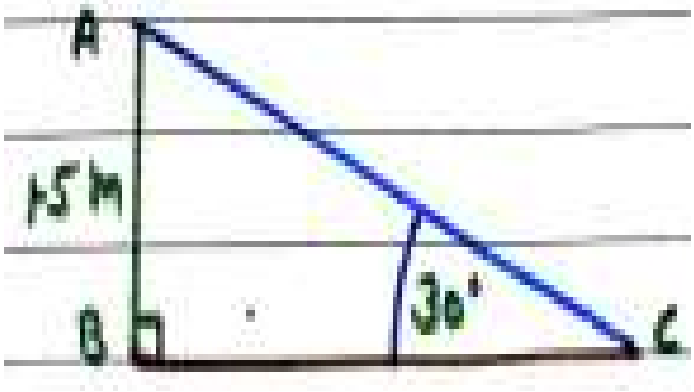
3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Sol. Let AC is the length of the slide for the children below the age of 5 years.

In $\perp \Delta$ ABC, $\frac{AB}{AC} = \sin\theta$

$\Rightarrow AB = AC \times \sin 30°$

$\Rightarrow 1.5 = AC \times \frac{1}{2}$



$\Rightarrow 2 \times 1.5 = AC$

$\Rightarrow AC = 3$ m

And let PQ is the length of the slide for elder children.

In $\perp \Delta$ PQR, $\frac{PQ}{PR} = \sin\theta$

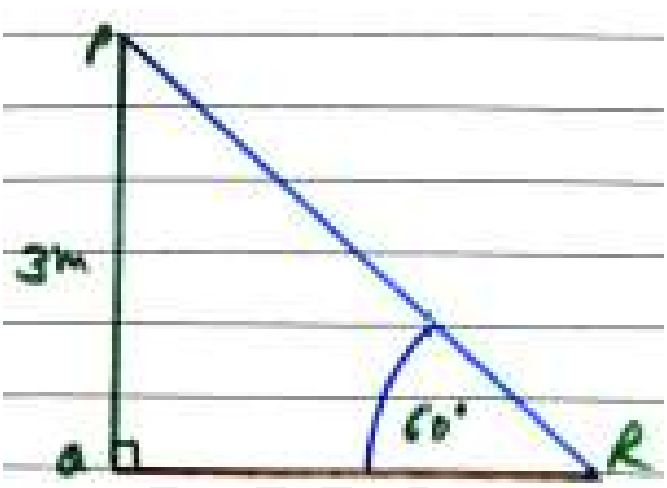
$\Rightarrow PQ = PR \times \sin 60°$

$\Rightarrow 3 = PR \times \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{6}{\sqrt{3}} = PR$

$\Rightarrow PR = \frac{6\sqrt{3}}{3}$

$\Rightarrow PR = 2\sqrt{3}$ m



∴ The length of the slide for the children below the age of 5 years is 3m and the length of the slide for elder children is $2\sqrt{3}$ m. Ans.

4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

Sol. Let AB is the height of tower.

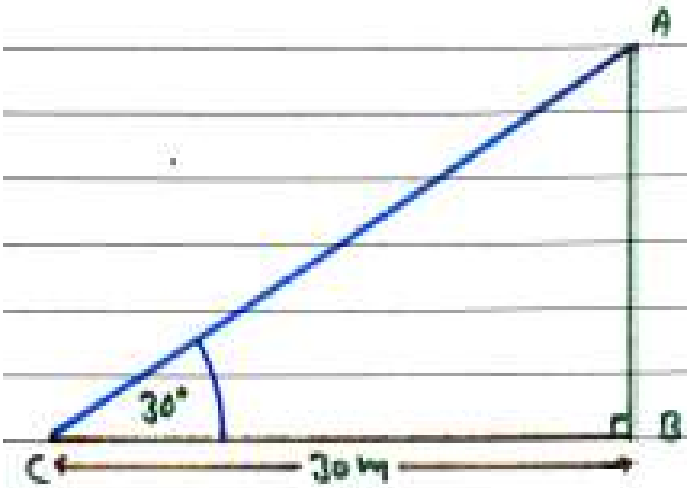
In $\perp \Delta$ ABC, $\frac{AB}{BC} = \tan\theta$

$\Rightarrow AB = BC \times \tan 30°$

$\Rightarrow AB = 30 \times \frac{1}{\sqrt{3}}$

$\Rightarrow AB = 30 \times \frac{\sqrt{3}}{3}$

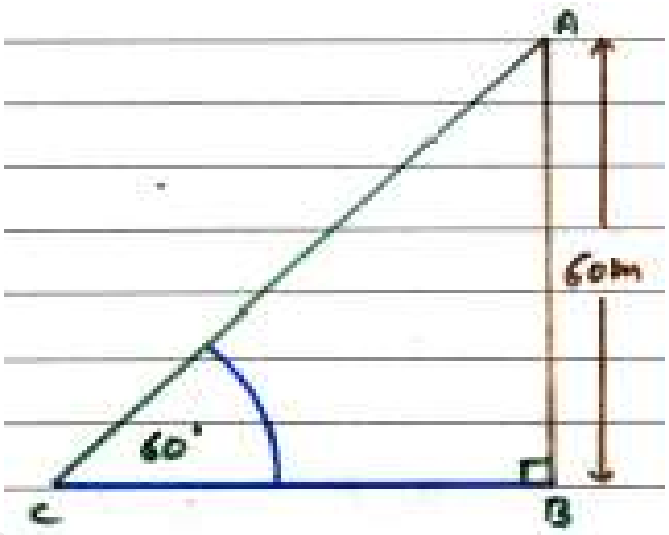
$\Rightarrow AB = 10\sqrt{3}$



∴ The height of the tower is $10\sqrt{3}$ m Ans.

5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Sol. Let AC is the length of the string.



In $\perp \Delta$ ABC, $\frac{AB}{AC} = \sin\theta$

$\Rightarrow AB = AC \times \sin 60°$

$\Rightarrow 60 = AC \times \frac{\sqrt{3}}{2}$

$\Rightarrow AC = 60 \times \frac{2}{\sqrt{3}}$

$\Rightarrow AC = 60 \times \frac{2\sqrt{3}}{3}$

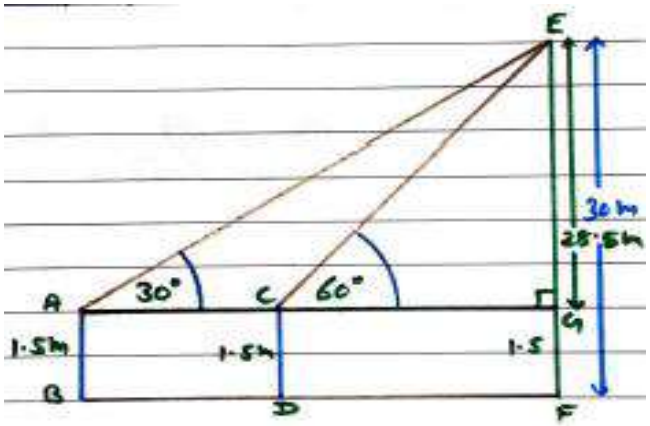
$\Rightarrow AC = 20 \times 2\sqrt{3}$

$\Rightarrow AC = 40\sqrt{3}$

$\therefore$ The length of the string is $40\sqrt{3}$ m. Ans.

6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Sol. Let AB is the height of boy and EF is the height of building. A and C are two observation points.



According to figure

AB = CD = GF = 1.5 m

AG = 30 m – 1.5 m

= 28.5 m

In $\perp \Delta$ ECG, $\frac{EG}{CG} = \tan\theta$

$\Rightarrow EG = CG \times \tan 60°$

$\Rightarrow 28.5 = CG \times \sqrt{3}$

$\Rightarrow CG = 28.5 \times \frac{1}{\sqrt{3}}$

$\Rightarrow CG = \frac{28.5\sqrt{3}}{3}$ m

$\Rightarrow CG = 9.5\sqrt{3}$ m

And in $\perp \Delta$ EAG, $\frac{EG}{AG} = \tan\theta$

$\Rightarrow EG = AG \times \tan 30°$

$\Rightarrow 28.5 = AG \times \frac{1}{\sqrt{3}}$

$\Rightarrow AG = 28.5 \times \sqrt{3}$ m

$\Rightarrow AC = AG - CG$

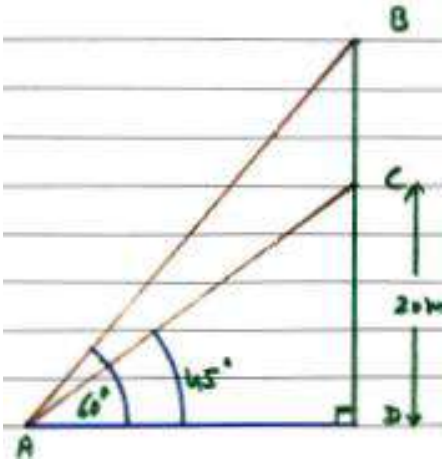
$= 28.5\sqrt{3}$ m - $9.5\sqrt{3}$ m

$= 19\sqrt{3}$ m

$\therefore$ The distance he walked towards the building is $19\sqrt{3}$ m Ans.

7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Sol. Let CD is building of height 20 m and BC is the height of tower. A is the observation point.



In $\perp \Delta$ ACD, $\frac{CD}{AD} = \tan\theta$

$\Rightarrow CD = AD \times \tan 45°$

$\Rightarrow 20 = AD \times 1$

$\Rightarrow AD = 20$

And in $\perp \Delta$ ABD, $\frac{BD}{AD} = \tan\theta$

$\Rightarrow BD = AD \times \tan 60°$

$\Rightarrow BD = 20 \times \tan 60°$

$\Rightarrow BD = 20 \times \sqrt{3}$

$\Rightarrow BD = 20\sqrt{3}$

Now, $BC = BD - CD$

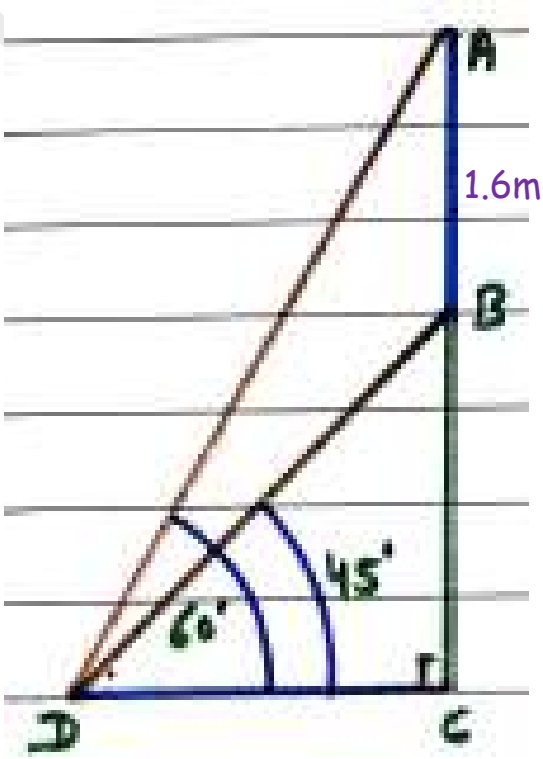
$= 20\sqrt{3} - 20$

$= 20(\sqrt{3} - 1)$

∴ The height of the tower $20(\sqrt{3} - 1)$ m Ans.

8. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Sol. Let AB is statue of height 1.6m and BC is the height of pedestal. D is the observation point.



In $\perp \Delta$ DBC, $\frac{BC}{DC} = \tan\theta$

$\Rightarrow BC = DC \times \tan 45°$

$\Rightarrow BC = DC \times 1$

$\Rightarrow BC = DC = x\ (say)$

And in $\perp \Delta$ ADC, $\frac{AC}{DC} = \tan\theta$

$\Rightarrow AC = DC \times \tan 60°$

$\Rightarrow AB + BC = DC \times \sqrt{3}$

$\Rightarrow 1.6 + x = x \times \sqrt{3}$

$\Rightarrow 1.6 = \sqrt{3}\,x - x$

$\Rightarrow 1.6 = (\sqrt{3} - 1)x$

$\Rightarrow x = \frac{16}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$= \frac{16(\sqrt{3}+1)}{3-1}$

$= 0.8(\sqrt{3}+1)$

∴ The height of the pedestal $0.8(\sqrt{3}+1)$ m Ans.

9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

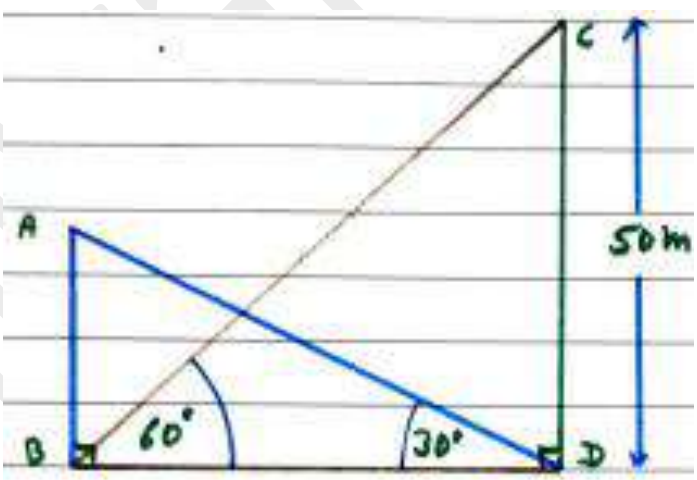
Sol. Let AB is the height of building and CD is the tower of height 50m.

According to the figure in ⊥ Δ BDC, $\frac{CD}{BD} = \tan\theta$

$\Rightarrow CD = BD \times \tan 60°$

$\Rightarrow 50 = BD \times \sqrt{3}$

$\Rightarrow BD = \frac{50}{\sqrt{3}}$ ..... (1)



And in ⊥ Δ ABD, $\frac{AB}{BD} = \tan\theta$

$\Rightarrow AB = BD \times \tan 30°$

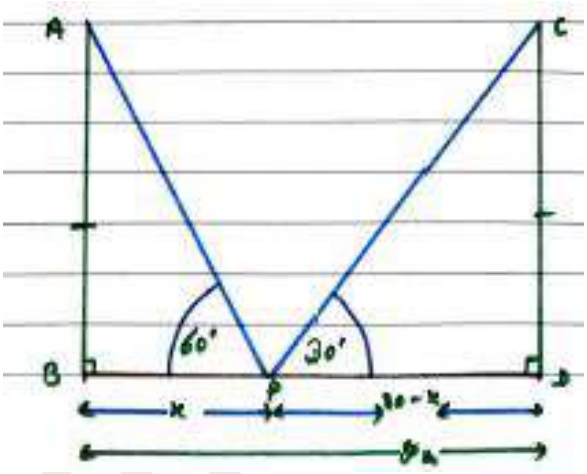
$\Rightarrow AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$ from (1)

$\Rightarrow AB = \frac{50}{3}$ m

∴ The height of the building is $16\frac{2}{3}$ m Ans.

10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

Sol. Let AB and CD are poles of equal height and P is the observation point.



In $\perp \Delta$ ABP, $\frac{AB}{BP} = \tan\theta$

$\Rightarrow AB = BP \times \tan 60°$

$\Rightarrow AB = BP \times \sqrt{3}$

$\Rightarrow AB = \sqrt{3}\, BP$ ..... (i)

In $\perp \Delta$ CDP, $\frac{CD}{DP} = \tan\theta$

$\Rightarrow CD = DP \times \tan 30°$

$\Rightarrow CD = (80 - BP) \times \frac{1}{\sqrt{3}}$

$\Rightarrow CD = (80 - BP)\frac{1}{\sqrt{3}}$ ..... (ii)

Since AB = CD

$\Rightarrow \sqrt{3}\, BP = (80 - BP)\frac{1}{\sqrt{3}}$

$\Rightarrow 3\, BP = 80 - BP$

$\Rightarrow 4\, BP = 80$

$\Rightarrow BP = 20$

Put the value of BP in equation (i), we have

$AB = 20\sqrt{3}$

$\therefore$ The Height of each pole = $20\sqrt{3}$ m.

The point is at a distance of 20 m from pole AB and at a distance of 60 m from pole CD. Ans.

11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

Sol. Let AB is the tower. C and D are two observation points.

In $\perp \Delta$ ABC, $\frac{AB}{BC} = \tan\theta$

$\Rightarrow AB = BC \times \tan 60°$

$\Rightarrow AB = BC \times \sqrt{3}$

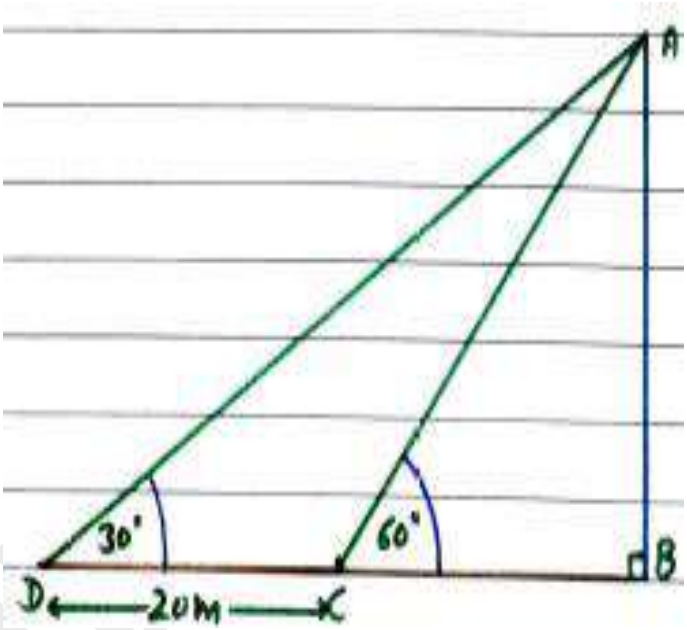
$\Rightarrow AB = \sqrt{3}\, BC$ ..... (i)

And in $\perp \Delta$ ABD, $\frac{AB}{BD} = \tan\theta$

$\Rightarrow AB = BD \times \tan 30°$

$\Rightarrow AB = (BC + CD) \times \frac{1}{\sqrt{3}}$

$\Rightarrow AB = (BC + 20)\frac{1}{\sqrt{3}}$..... (ii)



From (i) and (ii), we have

$\sqrt{3}\, BC = (BC + 20)\frac{1}{\sqrt{3}}$

$3\, BC = BC + 20$

$2\, BC = 20$

$BC = 10$

Put the value of BC in equation (i), we have

AB = $10\sqrt{3}$

$\therefore$ The height of tower is $10\sqrt{3}$ m and the width of canal is 10 m. Ans.

12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Sol. Let AB is the building of height 7m and CD is the height of tower. According to the figure in $\perp$ triangle ABD,

$\frac{AB}{BD} = \tan\theta$

$\Rightarrow AB = BD \times \tan 45°$

$\Rightarrow 7 = BD \times 1$

$\Rightarrow BD = 7$

$\Rightarrow BD = AE = 7$ m ..... (1)

And in $\perp \Delta$ AEC, $\frac{CE}{AE} = \tan\theta$

$\Rightarrow CE = AE \times \tan 60°$

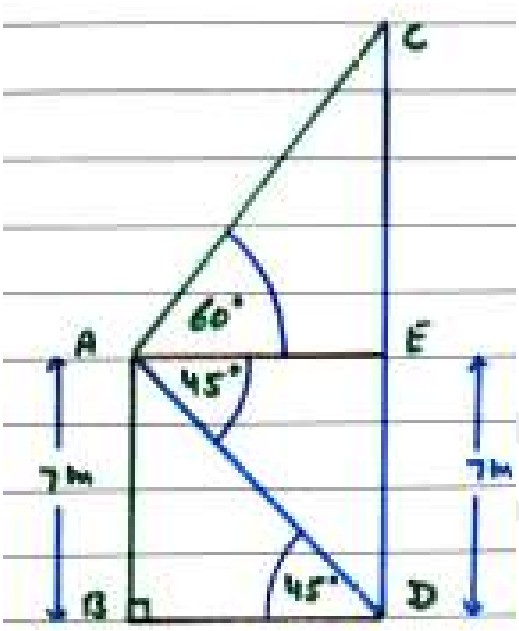
$\Rightarrow CE = 7 \times \sqrt{3}$ from (1)

$\Rightarrow CE = 7\sqrt{3}$ m

Now CD = CE + ED

$= 7\sqrt{3} + 7$

$= 7(\sqrt{3} + 1)$ m



∴ The height of the tower is $7(\sqrt{3} + 1)$ m Ans.

13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Sol. Let AB is lighthouse of height 75 m. C and D are the two positions of both ships and A is the observation point.

In $\perp \Delta$ ABC, $\frac{AB}{BC} = \tan\theta$

$\Rightarrow AB = BC \times \tan 45°$

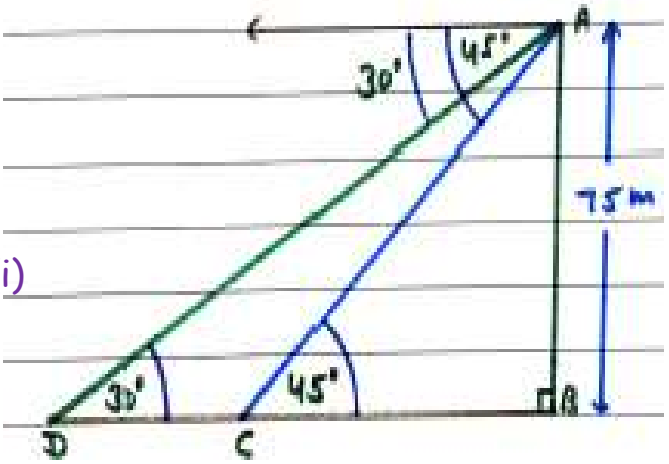
$\Rightarrow AB = BC \times 1$

$\Rightarrow AB = BC = 75\ m$ ..... (i)

And in $\perp \Delta$ ABD, $\frac{AB}{BD} = \tan\theta$

$\Rightarrow AB = BD \times \tan 30°$

$\Rightarrow 75 = (BC + CD) \times \frac{1}{\sqrt{3}}$



$\Rightarrow 75\sqrt{3} = 75 + CD$

$\Rightarrow CD = 75(\sqrt{3} - 1)$

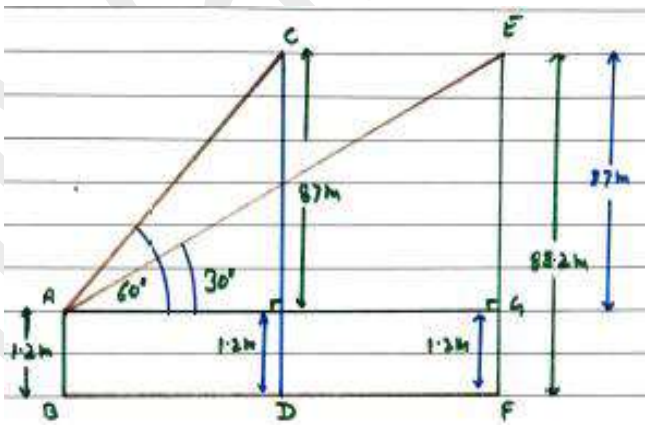
∴ The distance between two ships is 75($\sqrt{3}$ - 1) m Ans.

14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (see Fig. 9.13). Find the distance travelled by the balloon during the interval.

Sol. Let AB is the height of girl. CD and EF are the height of balloon. Bis the observation point.

Here CG =CD – GD = 88.2 – 1.2 = 87 m

In ⊥ Δ ACG, $\frac{CG}{AG} = \tan\theta$



$\Rightarrow CG = AG \times \tan 60°$

$\Rightarrow 87 = AG \times \sqrt{3}$

$\Rightarrow AG = \frac{87}{\sqrt{3}}$

$\Rightarrow AG = \frac{87\sqrt{3}}{3}$

$\Rightarrow AG = 29\sqrt{3}$ ..... (i)

And in ⊥ Δ AEH, $\frac{EH}{AH} = \tan\theta$

$\Rightarrow EH = AH \times \tan 30°$

$\Rightarrow 87 = AH \times \frac{1}{\sqrt{3}}$

$\Rightarrow AH = 87\sqrt{3}$..... (ii)

From (i) and (ii), we have

GH = AH – AG

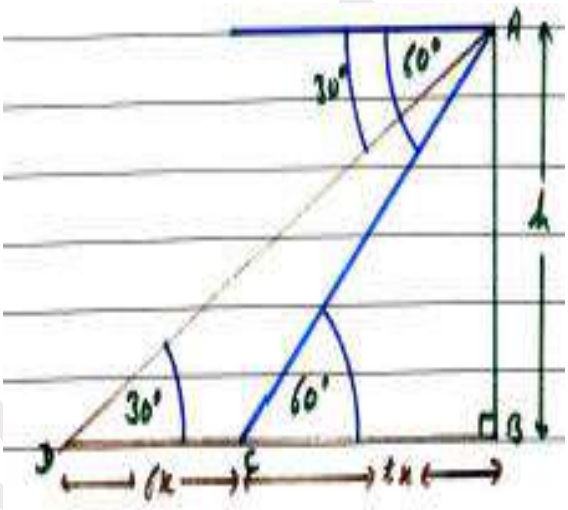
$= 87\sqrt{3} - 29\sqrt{3}$

$= 58\sqrt{3}$

∴ The distance travelled by the balloon during the interval is $58\sqrt{3}$ m. Ans.

15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Sol. Let AB is the height of the tower. C and D are the two positions of the same car. A is the observation point.



Let the speed of the car is x m/sec.

∴ distance covered in 6 seconds = 6x m

and distance covered in t seconds is tx m.

In $\perp \Delta$ ABC, $\frac{AB}{BC} = \tan\theta$

$\Rightarrow AB = BC \times \tan 60°$

$\Rightarrow AB = tx \times \sqrt{3}$

$\Rightarrow AB = tx\sqrt{3}\ m$ ..... (i)

And in $\perp \Delta$ ABD, $\frac{AB}{BD} = \tan\theta$

$\Rightarrow AB = BD \times \tan 30°$

$\Rightarrow tx\sqrt{3} = (BC + CD) \times \tan 30°$ [from ..... (i)]

$\Rightarrow tx\sqrt{3} = (tx + CD) \times \frac{1}{\sqrt{3}}$

$\Rightarrow 3tx = tx + 6x$

$\Rightarrow 3t = t + 6$

$\Rightarrow 2t = 6$

$\Rightarrow t = 3$

∴ Time taken by the car to reach the foot of the tower is 3 seconds. Ans.